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A DIFFERENCE-INDEX BASED RANKING METHOD OF TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS AND APPLICATION TO MULTIATTRIBUTE DECISION MAKING

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Abstract- The order relation of fuzzy number is important in decision making and optimization modeling, and ranking fuzzy numbers is difficult in nature. Ranking trapezoidal intuitionistic fuzzy numbers (TrIFNs) is more difficult due to the fact that the TrIFNs are a generalization of the fuzzy numbers. The aim of this paper is to develop a new methodology for ranking TrIFNs. We define the value-index and ambiguity-index based on the value and ambiguity of the membership and non-membership functions, and then propose a difference-index based ranking method, which is applied to multiattribute decision making (MADM) problems. The proposed method is compared to show its advantages and applicability.

Keywords- Trapezoidal intuitionistic fuzzy number, fuzzy set, ranking method, fuzzy decision making

1. INTRODUCTION

There is always existing uncertainty and imprecision in real-life decision making, the concept of the intuitionistic fuzzy (IF) set (IFS), introduced by Atanassov[1], is considered as a representation for these uncertain factors in real-life decision situations. Trapezoidal intuitionistic fuzzy numbers (TrIFNs) are special cases of IFSs defined on the set of real numbers, which may deal with more ill-known quantities, knowledge or experience. So TrIFNs play an important role in decision making and optimization modeling [2-4].

Different ranking methods of fuzzy numbers maybe produce different ranking results, which can bring some difficulties for decision makers. In addition, ranking fuzzy numbers is difficult in nature, especially the ranking methods of IF and IFS. Nowadays, there are some researches on the field. Nayagam et al. [5] described a type of special IFNs and introduced a scoring method of the special IFNs, which is a generalization of the scoring method for ranking fuzzy numbers. Zhang and Xu [6]propose a new method for ranking intuitionistic fuzzy values (IFVs) by using the similarity measure and the accuracy degree.Dymova et al. [7]proposed a new approach to estimate the strength of relations between real-valued and interval-valued IF values by the score and accuracy functions. Shu et al. (2006) developed an algorithm of the IF fault tree analysis for triangular IF numbers (TIFNs). Li [9] proposed a ratio ranking method of TIFNs based on the concept of value-index and ambiguity-index. Li et al. [10]

proposed a value and ambiguity based ranking method through defining the values and ambiguities of the membership and non-membership degrees for TIFNs.Wang and Zhang [2] defined the TrIFNs and gave a ranking method, which transformed the ranking of TrIFNs into the ranking of interval numbers.

From the existing research results, we can see that there exists little investigation on the ranking of TrIFNs. In addition, the TrIFNs are a generalization of IF numbers, and which are commonly used in real decision problems with the lack of information or imprecision of the available information in real situations is more serious. So the research of ranking TrIFNs is very necessary. However, the ranking problem is more difficult than ranking fuzzy numbers due to additional non-membership functions[7-14].The possibility value and possibility ambiguity are the important mathematical characteristics of fuzzy numbers. Therefore, introducing the value-index and ambiguity-index based ranking method is developed for TrIFNs and used in MADM problems. Compared with the existing research, the proposed method has a natural appealing interpretation and possesses some good properties such as the linearity , as well as it is more easily to be handled and calculated. And the method can be extended to more general IFNs. So the proposed method is of a great importance for scientific researches and real applications.

This paper is organized as follows. In Section 2, the concepts of TrIFNs and arithmetical operations as well as cut sets are introduced. Section 3 defines the concepts of value-index and ambiguity-index based on the value and ambiguity of the membership and non-membership functions. Hereby a difference-index based ranking method is developed. Section 4 formulates MADM problems with TrIFNs, which is solved by using the extended simple weighted average method according to the proposed ranking method. A numerical example and comparison analysis are given in Section 5. Section 6 contains the conclusion.

2. TRIFNS AND CUT SETS

2.1. The definition and operations of TrIFNs

A TrIFN $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ is a special IFS on a real number set R, whose membership function and non-membership function are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-\underline{a})w_{\tilde{a}} / (a_1 - \underline{a}) & (\underline{a} \le x < a_1) \\ w_{\tilde{a}} & (a_1 \le x \le a_2) \\ (\overline{a}-x)w_{\tilde{a}} / (\overline{a} - a_2) & (a_2 < x \le \overline{a}) \\ 0 & (x < \underline{a}, x > \overline{a}) \end{cases} \tag{1}$$

and

$$\upsilon_{\tilde a}(x)=\begin{cases}[a_1-x+u_{\tilde a}(x-\underline{a})]/(a_1-\underline{a}) & (\underline{a}\le x<a_1)\\ u_{\tilde a} & (a_1\le x\le a_2)\\ [x-a_2+u_{\tilde a}(\overline{a}-x)]/(\overline{a}-a_2) & (a_2<x\le \overline{a})\\ 1 & (x<\underline{a},\ x>\overline{a})\end{cases} \tag{2}$$

respectively, depicted as in Fig. 1. $w_{\tilde a}$ and $u_{\tilde a}$ respectively represent the maximum membership degree and minimum non-membership degree so that they satisfy the conditions: $0\le w_{\tilde a}\le 1$, $0\le u_{\tilde a}\le 1$ and $0\le w_{\tilde a}+u_{\tilde a}\le 1$. $\pi_{\tilde a}(x)=1-\mu_{\tilde a}(x)-\upsilon_{\tilde a}(x)$ is an IF index of an element x in $\tilde a$.

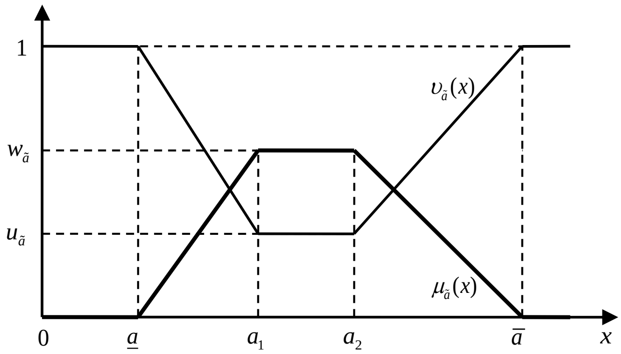

Figure 1. A TrIFN $\tilde a=<(\underline{a},a_1,a_2,\overline{a});w_{\tilde a},u_{\tilde a}>$

A TrIFN $\tilde a=<(\underline{a},a_1,a_2,\overline{a});w_{\tilde a},u_{\tilde a}>$ may express an approximate range of a closed interval $[a_1,a_2]$, which is approximately equal to $[a_1,a_2]$. Namely, the ill-known quantity "approximate $[a_1,a_2]$" is expressed using any value between $\underline{a}$ and $\overline{a}$ with different membership and non-membership degrees. Where $\mu_{\tilde a}(x)+\upsilon_{\tilde a}(x)=1$ for any $x\in \mathrm{R}$ if $w_{\tilde a}=1$ and $u_{\tilde a}=0$. Hence, the TrIFN $\tilde a=<(\underline{a},a_1,a_2,\overline{a});w_{\tilde a},u_{\tilde a}>$ degenerates to $\tilde a=<(\underline{a},a_1,a_2,\overline{a});1,0>$, which is just about a trapezoid fuzzy number [15]. On the other hand, if $b=c$ then $\tilde A=<(a,b,c,d);w_{\tilde A},u_{\tilde A}>$ degenerates to $\tilde A=<(a,b,d);w_{\tilde A},u_{\tilde A}>$, which is a TIFN.

For any TrIFNs $\tilde a=<(\underline{a},a_1,a_2,\overline{a});w_{\tilde a},u_{\tilde a}>$ and $\tilde b=<(\underline{b},b_1,b_2,\overline{b});w_{\tilde b},u_{\tilde b}>$, we stipulate the arithmetical operations as follows:

$$\tilde a+\tilde b=<(\underline{a}+\underline{b},a_1+b_1,a_2+b_2,\overline{a}+\overline{b});w_{\tilde a}\wedge w_{\tilde b},u_{\tilde a}\vee u_{\tilde b}> \tag{3}$$

$$\tilde a-\tilde b=<(\underline{a}-\overline{b},a_1-b_2,a_2-b_1,\overline{a}-\underline{b});w_{\tilde a}\wedge w_{\tilde b},u_{\tilde a}\vee u_{\tilde b}> \tag{4}$$

$$\tilde a\tilde b=\tilde a\times\tilde b=\begin{cases}<(\underline{a}\underline{b},a_1b_1,a_2b_2,\overline{a}\overline{b});w_{\tilde a}\wedge w_{\tilde b},u_{\tilde a}\vee u_{\tilde b}> & (\tilde a>0,\ \tilde b>0)\\ <(\underline{a}\overline{b},a_1b_2,a_2b_1,\overline{a}\underline{b});w_{\tilde a}\wedge w_{\tilde b},u_{\tilde a}\vee u_{\tilde b}> & (\tilde a<0,\ \tilde b>0)\\ <(\overline{a}\overline{b},a_2b_2,a_1b_1,\underline{a}\underline{b});w_{\tilde a}\wedge w_{\tilde b},u_{\tilde a}\vee u_{\tilde b}> & (\tilde a<0,\ \tilde b<0)\end{cases} \tag{5}$$

$$\tilde a/\tilde b=\tilde a\div\tilde b=\begin{cases}<(\underline{a}/\overline{b},a_1/b_2,a_2/b_1,\overline{a}/\underline{b});w_{\tilde a}\wedge w_{\tilde b},u_{\tilde a}\vee u_{\tilde b}> & (\tilde a>0,\ \tilde b>0)\\ <(\overline{a}/\overline{b},a_2/b_2,a_1/b_1,\underline{a}/\underline{b});w_{\tilde a}\wedge w_{\tilde b},u_{\tilde a}\vee u_{\tilde b}> & (\tilde a<0,\ \tilde b>0)\\ <(\overline{a}/\underline{b},a_2/b_1,a_1/b_2,\underline{a}/\overline{b});w_{\tilde a}\wedge w_{\tilde b},u_{\tilde a}\vee u_{\tilde b}> & (\tilde a<0,\ \tilde b<0)\end{cases} \tag{6}$$

$$\gamma\tilde{a}=\begin{cases}<(\gamma\underline{a},\gamma a_1,\gamma a_2,\gamma\overline{a});w_{\tilde{a}},u_{\tilde{a}}> & \text{if } \gamma>0\\ <(\gamma\overline{a},\gamma a_2,\gamma a_1,\gamma\underline{a});w_{\tilde{a}},u_{\tilde{a}}> & \text{if } \gamma<0\end{cases} \tag{7}$$

$$\tilde{a}^{-1}=<(1/\overline{a},1/a_2,1/a_1,1/\underline{a});w_{\tilde{a}},u_{\tilde{a}}> \quad (\tilde{a}\neq 0) \tag{8}$$

where the symbols "$\wedge$" and "$\vee$" are the min and max operators, respectively.

2.2 Cut sets of TrIFNs

A α-cut set of a TrIFN $\tilde{a}$ is a crisp subset of R, which can be expressed as $\tilde{a}_\alpha=\{x\,|\,\mu_{\tilde{a}}(x)\geq\alpha\}$, where $0\leq\alpha\leq w_{\tilde{a}}$. A β-cut set of a TrIFN $\tilde{a}$ is a crisp subset of R, which can be expressed as $\tilde{a}_\beta=\{x\,|\,\upsilon_{\tilde{a}}(x)\leq\beta\}$, where $u_{\tilde{a}}\leq\beta\leq 1$. And it directly follows from membership and non-membership functions of a TrIFN that $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ are closed intervals, denoted by $\tilde{a}_\alpha=[L_{\tilde{a}}(\alpha),R_{\tilde{a}}(\alpha)]$ and $\tilde{a}_\beta=[L_{\tilde{a}}(\beta),R_{\tilde{a}}(\beta)]$, which can be calculated as follows:

$$\tilde{a}_\alpha=[L_{\tilde{a}}(\alpha),R_{\tilde{a}}(\alpha)]=[\frac{(w_{\tilde{a}}-\alpha)\underline{a}+\alpha a_1}{w_{\tilde{a}}},\frac{(w_{\tilde{a}}-\alpha)\overline{a}+\alpha a_2}{w_{\tilde{a}}}] \tag{9}$$

and

$$\tilde{a}_\beta=[L_{\tilde{a}}(\beta),R_{\tilde{a}}(\beta)]=[\frac{(1-\beta)a_1+(\beta-u_{\tilde{a}})\underline{a}}{1-u_{\tilde{a}}},\frac{(1-\beta)a_2+(\beta-u_{\tilde{a}})\overline{a}}{1-u_{\tilde{a}}}]. \tag{10}$$

3. A DIFFERENCE-INDEX BASED RANKING METHOD

3.1. Value and ambiguity of a TrIFN

The values of the membership and non-membership functions for a TrIFN $\tilde{a}=<(\underline{a},a_1,a_2,\overline{a});w_{\tilde{a}},u_{\tilde{a}}>$ are defined as follows:

$$V_\mu(\tilde{a})=\int_0^{w_{\tilde{a}}}\frac{L_{\tilde{a}}(\alpha)+R_{\tilde{a}}(\alpha)}{2}f(\alpha)\mathrm{d}\alpha \tag{11}$$

and

$$V_\upsilon(\tilde{a})=\int_{u_{\tilde{a}}}^1\frac{L_{\tilde{a}}(\beta)+R_{\tilde{a}}(\beta)}{2}g(\beta)\mathrm{d}\beta. \tag{12}$$

respectively, where $f(\alpha)$ is a non-negative and non-decreasing function on the interval $[0,w_{\tilde{a}}]$ with $f(0)=0$ and $f(w_{\tilde{a}})=1$; $g(\beta)$ is a non-negative and non-increasing function on the interval $[u_{\tilde{a}},1]$ with $g(u_{\tilde{a}})=1$ and $g(1)=0$. Obviously, $f(\alpha)$ and $g(\beta)$ can be considered as weighting functions, and have various specific forms in actual applications, which can be chosen according to the real-life situations. Here,

$$f(\alpha)=\alpha/w_{\tilde{a}} \quad (\alpha\in[0,w_{\tilde{a}}]) \tag{13}$$

and

$$g(\beta)=(1-\beta)/(1-u_{\tilde{a}}) \quad (\beta\in[u_{\tilde{a}},1]). \tag{14}$$

The function $f(\alpha)$ gives different weights to elements at different α-cuts so that it can lessen the contribution of the lower α-cuts, since these cuts arising from values of $\mu_{\tilde{a}}(x)$ have a considerable amount of uncertainty. Therefore, $V_\mu(\tilde{a})$ and $V_\upsilon(\tilde{a})$ synthetically reflects the information on membership and non-membership degrees.

According to Eqs. (9), (11) and (13), the value of the membership function of a TrIFN $\tilde{a}$ is calculated as follows:

$$V_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} [\frac{(w_{\tilde{a}}-\alpha)\underline{a}+\alpha a_1}{2w_{\tilde{a}}} + \frac{(w_{\tilde{a}}-\alpha)\overline{a}+\alpha a_2}{2w_{\tilde{a}}}]\frac{\alpha}{w_{\tilde{a}}}\mathrm{d}\alpha$$
$$= [\frac{(\underline{a}+\overline{a})\alpha^2}{4w_{\tilde{a}}} + \frac{(a_1+a_2-\underline{a}-\overline{a})\alpha^3}{6(w_{\tilde{a}})^2}]|_0^{w_{\tilde{a}}} = \frac{(\underline{a}+2a_1+2a_2+\overline{a})w_{\tilde{a}}}{12}. \tag{15}$$

In a similar way, according to Eqs. (10), (12) and (14), the value of the non-membership function can be obtained as follows:

$$V_\upsilon(\tilde{a}) = \frac{(\underline{a}+2a_1+2a_2+\overline{a})(1-u_{\tilde{a}})}{12}. \tag{16}$$

It is directly derived from the condition $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$ that $0 \le V_\mu(\tilde{a}) \le V_\upsilon(\tilde{a})$, which may be concisely expressed as an interval $[V_\mu(\tilde{a}), V_\upsilon(\tilde{a})]$. Thus, $V_\mu(\tilde{a})$ and $V_\upsilon(\tilde{a})$ have some useful properties, which are summarized as in Theorems 1 and 2, respectively.

Theorem 1. Assume that $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ and $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$ are two TrIFNs with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$. Then, $V_\mu(\tilde{a}+\tilde{b}) = V_\mu(\tilde{a}) + V_\mu(\tilde{b})$.

Proof. According to Eq. (3) with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$, we have $\tilde{a}+\tilde{b} = <(\underline{a}+\underline{b}, a_1+b_1, a_2+b_2, \overline{a}+\overline{b}); w_{\tilde{a}}, u_{\tilde{a}}>$. Using Eq. (15), we obtain

$$V_\mu(\tilde{a}+\tilde{b}) = \frac{[(\underline{a}+\underline{b})+2(a_1+b_1)+2(a_2+b_2)+(\overline{a}+\overline{b})]w_{\tilde{a}}}{12}$$
$$= \frac{(\underline{a}+2a_1+2a_2+\overline{a})w_{\tilde{a}}^2}{12} + \frac{(\underline{b}+2b_1+2b_2+\overline{b})w_{\tilde{b}}}{12} = V_\mu(\tilde{a}) + V_\mu(\tilde{b})$$

Thus, Theorem 1 has been proven.

In the same way to Theorem 1, we can prove Theorem 2 as follows. .

Theorem 2. Assume that $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ and $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$ are two TrIFNs with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$. Then, $V_\upsilon(\tilde{a}+\tilde{b}) = V_\upsilon(\tilde{a}) + V_\upsilon(\tilde{b})$.

The ambiguities of the membership and non-membership functions for a TrIFN $\tilde{a}$ are defined as follows:

$$A_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} (R_{\tilde{a}}(\alpha) - L_{\tilde{a}}(\alpha)) f(\alpha)\mathrm{d}\alpha \tag{17}$$

and

$$A_\upsilon(\tilde{a}) = \int_{u_{\tilde{a}}}^1 (R_{\tilde{a}}(\beta) - L_{\tilde{a}}(\beta)) g(\beta)\mathrm{d}\beta \tag{18}$$

$R_{\tilde{a}}(\alpha) - L_{\tilde{a}}(\alpha)$ and $R_{\tilde{a}}(\beta) - L_{\tilde{a}}(\beta)$ are just about the lengths of the intervals $\tilde{a}_\alpha$ and $\tilde{a}_\beta$, respectively. Thus, $A_\mu(\tilde{a})$ and $A_\upsilon(\tilde{a})$ may be regarded as the "global spreads" of the membership and the non-membership functions. Obviously, $A_\mu(\tilde{a})$ and $A_\upsilon(\tilde{a})$ basically measure how much there is uncertainty in $\tilde{a}$. obviously, $A_\mu(\tilde{a}) \ge 0$ and $A_\upsilon(\tilde{a}) \ge 0$.

According to Eqs. (9), (13) and (17), the ambiguity of the membership function of a TrIFN $\tilde{a}$ is calculated as follows:

$$A_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}} [\frac{(w_{\tilde{a}}-\alpha)\overline{a}+\alpha a_2}{w_{\tilde{a}}} - \frac{(w_{\tilde{a}}-\alpha)\underline{a}+\alpha a_1}{w_{\tilde{a}}}]\frac{\alpha}{w_{\tilde{a}}}\mathrm{d}\alpha$$
$$= [\frac{(\overline{a}-\underline{a})\alpha^2}{2w_{\tilde{a}}} - \frac{2(\overline{a}-\underline{a}+a_1-a_2)\alpha^3}{6(w_{\tilde{a}})^2}]|_0^{w_{\tilde{a}}} = \frac{(\overline{a}-\underline{a}+2a_2-2a_1)w_{\tilde{a}}}{6} \tag{19}$$

Likewise, according to Eqs. (10), (14) and (18), the ambiguity of the non-membership function of a TrIFN $\tilde{a}$ is calculated as follows:

$$A_\upsilon(\tilde{a}) = \frac{(\overline{a}-\underline{a}+2a_2-2a_1)(1-u_{\tilde{a}})}{6} \tag{20}$$

It is noted that $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$. Hence, $A_\mu(\tilde{a}) \le A_\upsilon(\tilde{a})$. Thus, the ambiguities of the membership and non-membership functions of a TrIFN $\tilde{a}$ can be expressed as an interval $[A_\mu(\tilde{a}), A_\upsilon(\tilde{a})]$. $A_\mu(\tilde{a})$ and $A_\upsilon(\tilde{a})$ have some useful properties, which are summarized as in Theorems 3 and 4, respectively.

Theorem 3. Assume that $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ and $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$ are two TrIFNs with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$. Then, $A_\mu(\tilde{a}+\tilde{b}) = A_\mu(\tilde{a}) + A_\mu(\tilde{b})$.

Proof. According to Eq. (3) with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$, we have $\tilde{a}+\tilde{b} = <(\underline{a}+\underline{b}, a_1+b_1, a_2+b_2, \overline{a}+\overline{b}); w_{\tilde{a}}, u_{\tilde{a}}>$. Using Eq. (19), we obtain

$$A_\mu(\tilde{a}+\tilde{b}) = \frac{[(\overline{a}+\overline{b})-(\underline{a}+\underline{b})+2(a_2+b_2)-2(a_1+b_1)]w_{\tilde{a}}}{6}$$
$$= \frac{(\overline{a}-\underline{a}+2a_2-2a_1)w_{\tilde{a}}}{6} + \frac{(\overline{b}-\underline{b}+2b_2-2b_1)w_{\tilde{b}}}{6} = A_\mu(\tilde{a}) + A_\mu(\tilde{b})$$

Thus, Theorem 3 has been proven.

In the same way to Theorem 3, we can prove Theorem 4 as follows.

Theorem 4. Assume that $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ and $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$ are two TrIFNs with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$. Then, $A_\upsilon(\tilde{a}+\tilde{b}) = A_\upsilon(\tilde{a}) + A_\upsilon(\tilde{b})$.

3.2. The difference-index based ranking method

In this subsection, we propose a new ranking method based on the difference-index of the value-index to the ambiguity-index for a TrIFN.

A value-index and an ambiguity-index for a TrIFN $\tilde{a}$ are defined as follows:

$$V(\tilde{a},\lambda) = V_\mu(\tilde{a}) + \lambda(V_\upsilon(\tilde{a}) - V_\mu(\tilde{a})) \tag{21}$$

and

$$A(\tilde{a},\lambda) = A_\upsilon(\tilde{a}) - \lambda(A_\upsilon(\tilde{a}) - A_\mu(\tilde{a})) \tag{22}$$

respectively, where $\lambda \in [0,1]$ is a weight which represents the decision maker's preference information. $\lambda \in [0,1/2)$ shows that decision maker prefers to uncertainty or negative feeling; $\lambda \in (1/2,1]$ shows that the decision maker prefers to certainty or positive feeling; $\lambda = 1/2$ shows that the decision maker is indifferent between positive feeling and negative feeling. Therefore, the value-index and the ambiguity-index may reflect the decision maker's subjectivity attitude to the TrIFN.

Remark 1. It is easily seen that the value-index $V(\tilde{a},\lambda)$ should be maximized whereas the ambiguity-index $A(\tilde{a},\lambda)$ should be minimized. Furthermore, $V(\tilde{a},\lambda)$ and $A(\tilde{a},\lambda)$

have some useful properties, which are summarized as in Theorems 5-7, respectively.

Theorem 5. $V(\tilde{a},\lambda)$ and $A(\tilde{a},\lambda)$ are continuous non-decreasing and non-increasing functions of the parameter $\lambda \in [0,1]$, respectively.

Proof. It is known that $V(\tilde{a},\lambda)$ and $A(\tilde{a},\lambda)$ are linear functions of $\lambda \in [0,1]$. Hence, they are continuous functions of $\lambda \in [0,1]$. According to Eqs. (21) and (22), partial derivatives of $V(\tilde{a},\lambda)$ and $A(\tilde{a},\lambda)$ with respect to $\lambda \in [0,1]$ can be calculated as follows:

$$\frac{\mathrm{d}V(\tilde{a},\lambda)}{\mathrm{d}\lambda} = V_{\upsilon}(\tilde{a}) - V_{\mu}(\tilde{a}) \quad , \quad \frac{\mathrm{d}A(\tilde{a},\lambda)}{\mathrm{d}\lambda} = -A_{\upsilon}(\tilde{a}) + A_{\mu}(\tilde{a})$$

Noted that $V_{\upsilon}(\tilde{a}) \geq V_{\mu}(\tilde{a})$ and $A_{\upsilon}(\tilde{a}) \geq A_{\mu}(\tilde{a})$. Therefore, $\frac{\mathrm{d}V(\tilde{a},\lambda)}{\mathrm{d}\lambda} \geq 0$ and $\frac{\mathrm{d}A(\tilde{a},\lambda)}{\mathrm{d}\lambda} \leq 0$. Hence, $V(\tilde{a},\lambda)$ and $A(\tilde{a},\lambda)$ are non-decreasing and non-increasing functions of the parameter $\lambda \in [0,1]$, respectively.

Theorem 6. Assume that $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ and $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$ are two TrIFNs with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$. Then, $V(\tilde{a}+\tilde{b},\lambda) = V(\tilde{a},\lambda) + V(\tilde{b},\lambda)$, where $\lambda \in [0,1]$.

Proof. Using Eq. (21), we have $V(\tilde{a}+\tilde{b},\lambda) = V_{\mu}(\tilde{a}+\tilde{b}) + \lambda[V_{\upsilon}(\tilde{a}+\tilde{b}) - V_{\mu}(\tilde{a}+\tilde{b})]$. Combining with Theorems 1 and 2, we obtain

$$\begin{aligned} V(\tilde{a}+\tilde{b},\lambda) &= (V_{\mu}(\tilde{a}) + V_{\mu}(\tilde{b})) + \lambda[(V_{\upsilon}(\tilde{a}) + V_{\upsilon}(\tilde{b})) - (V_{\mu}(\tilde{a}) + V_{\mu}(\tilde{b}))] \\ &= [V_{\mu}(\tilde{a}) + \lambda(V_{\upsilon}(\tilde{a}) - V_{\mu}(\tilde{a}))] + [V_{\mu}(\tilde{b}) + \lambda(V_{\upsilon}(\tilde{b}) - V_{\mu}(\tilde{b}))] \\ &= V(\tilde{a},\lambda) + V(\tilde{b},\lambda) \end{aligned}$$

Thus, Theorem 6 has been proven.

Theorem 7. Assume that $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ and $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$ are two TrIFNs with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$. Then, $A(\tilde{a}+\tilde{b},\lambda) = A(\tilde{a},\lambda) + A(\tilde{b},\lambda)$, where $\lambda \in [0,1]$.

In the same way to Theorem 6, combining with Theorems 3 and 4, Theorem 7 can be easily proved.

A difference-index of a TrIFN $\tilde{a}$ is defined as follows:

$$\Delta(\tilde{a},\lambda) = V(\tilde{a},\lambda) - A(\tilde{a},\lambda) \tag{23}$$

Theorem 8. Assume that $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ and $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$ are two TrIFNs with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$. Then, $\Delta(\tilde{a}+\tilde{b},\lambda) = \Delta(\tilde{a},\lambda) + \Delta(\tilde{b},\lambda)$, where $\lambda \in [0,1]$.

Proof. According to Theorems 7 and 8, it is derived from Eq. (23) that

$$\begin{aligned} \Delta(\tilde{a}+\tilde{b},\lambda) &= V(\tilde{a}+\tilde{b},\lambda) - A(\tilde{a}+\tilde{b},\lambda) = (V(\tilde{a},\lambda) + V(\tilde{b},\lambda)) - (A(\tilde{a},\lambda) + A(\tilde{b},\lambda)) \\ &= (V(\tilde{a},\lambda) - A(\tilde{a},\lambda)) + (V(\tilde{b},\lambda) - A(\tilde{b},\lambda)) = \Delta(\tilde{A},\lambda) + \Delta(\tilde{b},\lambda). \end{aligned}$$

Thus, Theorem 8 has been proven.

Theorem 8 shows that the difference-index $\Delta(\tilde{A},\lambda)$ is a linear function of any TrIFNs. Furthermore, it is can be seen that the larger the difference-index the bigger the TrIFN. Thus, we propose the difference-index based ranking method of TrIFNs as follows.

Definition 1. Assume that $\lambda \in [0,1]$. For any TrIFNs $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$ and $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$, we stipulate:

(1) $\Delta(\tilde{a},\lambda) > \Delta(\tilde{b},\lambda)$ if and only if $\tilde{a}$ is bigger than $\tilde{b}$, denoted by $\tilde{a} > \tilde{b}$;

(2) $\Delta(\tilde{a},\lambda) = \Delta(\tilde{b},\lambda)$ if and only if $\tilde{a}$ is equal to $\tilde{b}$, denoted by $\tilde{a} = \tilde{b}$;

(3) $\Delta(\tilde{a},\lambda) \geq \Delta(\tilde{b},\lambda)$ if and only if $\tilde{a} > \tilde{b}$ or $\tilde{a} = \tilde{b}$.

The proposed ranking method satisfies five properties proposed by Wang and Kerre [16], which serve as the reasonable properties for the ordering of fuzzy quantities. In addition, it is a kind of two-index approaches, which aggregates both the value-index and ambiguity-index. Especially, this method satisfies the linearity.

Theorem 9. The difference-index based ranking method of TrIFNs has the following properties.

(P1) For a TrIFN $\tilde{a}$, then $\tilde{a} \geq \tilde{a}$;

(P2) For any TrIFNs $\tilde{a}$ and $\tilde{b}$, if $\tilde{a} \geq \tilde{b}$ and $\tilde{b} \geq \tilde{a}$, then $\tilde{a} = \tilde{b}$;

(P3) For any TrIFNs $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$, if $\tilde{a} \geq \tilde{b}$ and $\tilde{b} \geq \tilde{c}$, then $\tilde{a} \geq \tilde{c}$;

(P4) Assume that F_1 and F_2 are two arbitrary finite subsets of TrFNs. For any TrIFNs $\tilde{a} \in F_1 \cap F_2$ and $\tilde{b} \in F_1 \cap F_2$, then $\tilde{a} > \tilde{b}$ on F_1 if and only if $\tilde{b} > \tilde{a}$ on F_2;

(P5) For any TrIFNs $\tilde{a} = <(\underline{a}, a_1, a_2, \overline{a}); w_{\tilde{a}}, u_{\tilde{a}}>$, $\tilde{b} = <(\underline{b}, b_1, b_2, \overline{b}); w_{\tilde{b}}, u_{\tilde{b}}>$ and $\tilde{c} = <(\underline{c}, c_1, c_2, \overline{c}); w_{\tilde{c}}, u_{\tilde{c}}>$ with $w_{\tilde{a}} = w_{\tilde{b}}$ and $u_{\tilde{a}} = u_{\tilde{b}}$, if $\tilde{a} \geq \tilde{b}$, then $\tilde{a} + \tilde{c} \geq \tilde{b} + \tilde{c}$.

Proof. Using Definition 1 and Eq. (23), Theorem 9 can be proven in a similar way to that Wang and Kerre (2001) (omitted).

4. AN EXTENDED MADM METHOD USING THE DIFFERENCE-INDEX BASED RANKING METHOD

In this section, we will extend the simple weighted average method to solve the MADM problems with TrIFNs. Suppose that there exists an alternative set $A = \{A_1, A_2, \cdots, A_m\}$, which consists of m non-inferior alternatives from which the most preferred alternative has to be selected. Each alternative is assessed on n attributes. Denote the set of all attributes by $X = \{X_1, X_2, \cdots, X_n\}$. Assume that ratings of alternatives on attributes are expressed with TrIFNs. Namely, the rating of each alternative $A_i \in A$ ($i = 1, 2, \cdots, m$) on every attribute $X_j \in X$ ($j = 1, 2, \cdots, n$) is given as a TrIFN $\tilde{a}_{ij} = <(\underline{a}_{ij}, a_{1ij}, a_{2ij}, \overline{a}_{ij}); w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}}>$. Thus, an MADM problem with TrIFNs can be expressed concisely in the matrix format as $(\tilde{a}_{ij})_{m \times n}$. Due to the fact that attributes may have different importance degree. Assume that the relative weight of the attribute X_j is ω_j ($j = 1, 2, \cdots, n$), which should satisfy the normalization conditions: $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. Let $\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ be the relative weight vector of all attributes. The extended simple weighted average method for the MADM problems with TrIFNs can be summarized as follows:

(a) Normalize the TrIFN decision matrix. In order to eliminate the effect of different physical dimensions on the final decision making results, the normalized TrIFN decision matrix can be calculated using the following formulae:

$$\tilde{r}_{ij} =< (\frac{\underline{a}_{ij}}{\bar{a}_j^+}, \frac{a_{1ij}}{\bar{a}_j^+}, \frac{a_{2ij}}{\bar{a}_j^+}, \frac{\bar{a}_{ij}}{\bar{a}_j^+}); w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} > \quad (i = 1, 2, \cdots, m;\ j \in B) \tag{24}$$

and

$$\tilde{r}_{ij} =< (\frac{\underline{a}_j^-}{\bar{a}_{ij}}, \frac{\underline{a}_j^-}{a_{2ij}}, \frac{\underline{a}_j^-}{a_{1ij}}, \frac{\underline{a}_j^-}{\underline{a}_{ij}}); w_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} > \quad (i = 1, 2, \cdots, m;\ j \in C) \tag{25}$$

respectively, where B and C are the sets of benefit attributes and cost attributes, and $\bar{a}_j^+ = \max\{\bar{a}_{ij} \mid i = 1, 2, \cdots, m\}$ and $\underline{a}_j^- = \min\{\underline{a}_{ij} \mid i = 1, 2, \cdots, m\}$ $(j = 1, 2, \cdots, n)$. For convenience, all $\tilde{r}_{ij}$ ($i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$) are uniformly denoted by $\tilde{r}_{ij} =< (\underline{r}_{ij}, r_{1ij}, r_{2ij}, \bar{r}_{ij}); w_{\tilde{r}_{ij}}, u_{\tilde{r}_{ij}} >$, where $w_{\tilde{r}_{ij}} = w_{\tilde{a}_{ij}}$ and $u_{\tilde{r}_{ij}} = u_{\tilde{a}_{ij}}$.

(b) Construct the weighted normalized TrIFN decision matrix. Using Eq. (7), the weighted normalized TrIFN decision matrix can be calculated as $(\tilde{u}_{ij})_{m \times n}$, where

$$\tilde{u}_{ij} = \omega_j \tilde{r}_{ij} =< (\omega_j \underline{r}_{ij}, \omega_j r_{1ij}, \omega_j r_{2ij}, \omega_j \bar{r}_{ij}); w_{\tilde{r}_{ij}}, u_{\tilde{r}_{ij}} > \quad (i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n) \tag{26}$$

(c) Calculate the weighted comprehensive values of alternatives. Using Eq. (3), the weighted comprehensive values of alternatives A_i $(i = 1, 2, \cdots, m)$ are calculated as follows:

$$\tilde{S}_i = \sum_{j=1}^{n} \tilde{u}_{ij} =< (\sum_{j=1}^{n} \omega_j \underline{r}_{ij}, \sum_{j=1}^{n} \omega_j r_{1ij}, \sum_{j=1}^{n} \omega_j r_{2ij}, \sum_{j=1}^{n} \omega_j \bar{r}_{ij}); \min_{1 \le j \le n}\{w_{\tilde{r}_{ij}}\}, \max_{1 \le j \le n}\{u_{\tilde{r}_{ij}}\} > \quad (i = 1, 2, \cdots, m) \tag{27}$$

respectively. Obviously, $\tilde{S}_i$ $(i = 1, 2, \cdots, m)$ are TrIFNs.

(d) Rank alternatives. For a given weight $\lambda \in [0,1]$, using Eq. (23), we compute $\Delta(\tilde{S}_i, \lambda)$ for each alternative. The ranking order of the alternatives A_i $(i = 1, 2, \cdots, m)$ is generated according to the non-increasing order of the difference-indexes $\Delta(\tilde{S}_i, \lambda)$. The best alternative is the one with the largest difference-index, i.e., $\max\{\Delta(\tilde{S}_i, \lambda) \mid i = 1, 2, \cdots, m\}$.

5. APPLICATION AND COMPARISON ANALYSIS

In this section, an example for a multiattribute decision making problem of alternatives is used with the proposed method, and compared to show its advantages and applicability. Due to information of compared examples are expressed with TIFN, and TIFN is a special form of TrIFN, so we use TIFN in the numerical example.

5.1. A personnel selection problem and analysis process

The proposed decision method is illustrated with a personnel selection problem, which is adapted from [9] and [10]. Suppose that a software company desires to hire a system analyst. After preliminary screening, three candidates (i.e., alternatives) A_1, A_2 and A_3 remain for further evaluation. The decision making committee assesses the three candidates based on five attributes, including emotional steadiness (X_1), oral communication skill (X_2), personality (X_3), past experience (X_4) and self-confidence

(X_5). The ratings of the candidates with respect to attributes are given in Tab.1, where <(5.7,7.7,9.3);0.7,0.2> in the Table 1 is an TIFN which indicates that the mark of the candidate 1 A_1 with respect to the attribute X_1 is about 7.7 with the maximum satisfaction degree is 0.7, while the minimum non-satisfaction degree is 0.2. In other words, the hesitation degree is 0.1. Other TIFNs in Table 1 are explained similarly. Since the five attributes are benefit attributes, according to Eqs. (24) and (26), the weighted normalized TIFN decision matrix is obtained as in Table 2.

Table 1. The TIFN decision matrix

Candidates	Attributes				
	X_1	X_2	X_3	X_4	X_5
A_1	<(5.7,7.7,9.3); 0.7,0.2>	<(5,7,9); 0.6,0.3>	<(5.7,7.7,9); 0.8,0.1>	<(8.33,9.67,10); 0.6,0.4>	<(3,5,7); 0.6,0.3>
A_2	<(6.5,8.6,10); 0.4,0.5>	<(8,9,10); 0.6,0.3>	<(8.3,9.7,10); 0.7,0.2>	<(8,9,10); 0.6,0.3>	<(7,9,10); 0.6,0.2>
A_3	<(6.5,8.2,9.3); 0.8,0.1>	<(7,9,10); 0.7,0.2>	<(0,9,10); 0.5,0.2>	<(6,8,9); 0.6,0.2>	<(6.3,8.3,9.7); 0.7,0.2>

Table 2. The weighted normalized TIFN decision matrix

Candidates	Attributes				
	X_1	X_2	X_3	X_4	X_5
A_1	<(0.083,0.111,0134); 0.7,0.2>	<(0.15,0.21,0.27) ;0.6,0.3>	<(0.068,0.092,0.108); 0.8,0.1>	<(0.249,0.291,0.3); 0.6,0.4>	<(0.042,0.07,0.098); 0.6,0.3>
A_2	<(0.091,0.12,0.14); 0.4,0.5>	<(0.24,0.27,0.3); 0.6,0.3>	<(0.1,0.116,0.12); 0.7,0.2>	<(0.24,0.27,0.3); 0.6,0.3>	<(0.098,0.126,0.14); 0.6,0.2>
A_3	<(0.091,0.115,0.13); 0.8,0.1>	<(0.21,0.27,0.3); 0.7,0.2>	<(0.084,0.108,0.12); 0.5,0.2>	<(0.18,0.24,0.27); 0.6,0.2>	<(0.088,0.116,0.136); 0.7,0.2>

Combining with Eq. (27), the weighted comprehensive values of the candidates A_i ($i=1,2,3$) can be obtained as follows: $\tilde{S}_1 = <(0.592, 0.774, 0.910); 0.6, 0.4>$, $\tilde{S}_2 = <(0.769, 0.903, 1); 0.4, 0.5>$, $\tilde{S}_3 = <(0.653, 0.849, 0.956); 0.5, 0.2>$.

According to Eqs. (17), (18) and (21), the value-indexes of $\tilde{S}_1$, $\tilde{S}_2$ and $\tilde{S}_3$ can be obtained as follows:

$$V(\tilde{S}_1,\lambda)=0.2298,\ V(\tilde{S}_2,\lambda)=0.1794+0.0449\lambda,\ V(\tilde{S}_3,\lambda)=0.2085+0.1251\lambda .$$

In the same way, according to Eqs. (19), (20) and (22), the ambiguity-indexes of $\tilde{S}_1$, $\tilde{S}_2$ and $\tilde{S}_3$ can be obtained as follows:

$$A(\tilde{S}_1,\lambda)=0.0318,\ A(\tilde{S}_2,\lambda)=0.0193-0.0039\lambda,\ A(\tilde{S}_3,\lambda)=0.0404-0.0151\lambda .$$

According to Eq. (23), the difference-indexes of $\tilde{S}_1$, $\tilde{S}_2$ and $\tilde{S}_3$ are obtained as follows:

$$\Delta(\tilde{S}_1,\lambda)=0.198,\ \Delta(\tilde{S}_2,\lambda)=0.1601+0.0488\lambda,\ \Delta(\tilde{S}_3,\lambda)=0.1681+0.1402\lambda,$$

respectively, depicted as in Fig. 2.

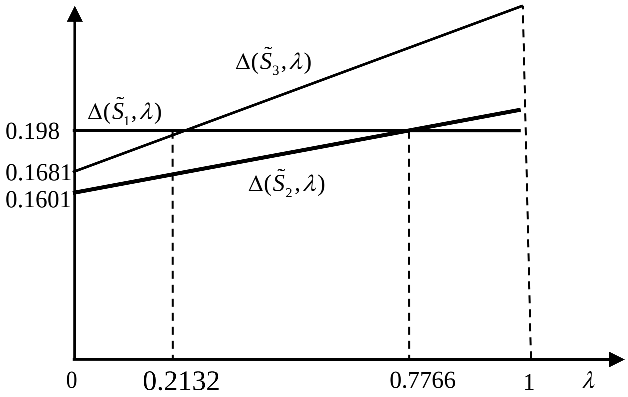

Figure 2. The difference-indexes of $\tilde{S}_1$, $\tilde{S}_2$ and $\tilde{S}_3$

It is easy to see from Fig. 2 that $\Delta(\tilde{S}_1,\lambda) > \Delta(\tilde{S}_3,\lambda) > \Delta(\tilde{S}_2,\lambda)$ for any given $\lambda \in [0, 0.2132)$, $\Delta(\tilde{S}_3,\lambda) > \Delta(\tilde{S}_1,\lambda) > \Delta(\tilde{S}_2,\lambda)$ for any given $\lambda \in (0.2132, 0.7766)$ and $\Delta(\tilde{S}_3,\lambda) > \Delta(\tilde{S}_2,\lambda) > \Delta(\tilde{S}_1,\lambda)$ for any given $\lambda \in (0.7772, 1]$. Hence, the ranking order of the three candidates is $A_1 \succ A_3 \succ A_2$ if $\lambda \in [0, 0.2132)$. However, if $\lambda \in (0.2132, 0.7766)$, the ranking order of the three candidates is $A_3 \succ A_1 \succ A_2$. If $\lambda \in (0.7766, 1]$, the ranking order is $A_3 \succ A_2 \succ A_1$. Obviously, the ranking order of the three candidates is related to the attitude parameter $\lambda \in [0,1]$.

5.2. Comparison analysis of the results obtained by the proposed method and other methods

(1) Comparison with Li's ranking method

Li [9] proposed a ratio ranking method for TIFN based on the value-index to the ambiguity-index. According to Li's method, the ranking order of the three candidates is generated as follows: $A_1 \succ A_3 \succ A_2$ if $\lambda \in [0, 0.1899)$, $A_3 \succ A_1 \succ A_2$ if $\lambda \in (0.1899, 0.9667)$, and $A_3 \succ A_2 \succ A_1$ if $\lambda \in (0.9667, 1]$. Li's method is related to the attitude parameter $\lambda \in [0,1]$, but it is not a linear function of a TIFN $\tilde{a}$ although both $V(\tilde{a},\lambda)$ and $A(\tilde{a},\lambda)$ are linear on $\tilde{a}$. In other words, $R(\tilde{a}+\tilde{b},\lambda) \neq R(\tilde{a},\lambda) + R(\tilde{b},\lambda)$.

(2) Comparison with Li and Nan' ranking method

Li and Nan [10]proposed a ranking method based on the value and ambiguity, which is essentially a lexicographical ranking method. According to Li and Nan's method, the ranking order of $\tilde{a}$ and $\tilde{b}$ depends on the relative position of $V(\tilde{a},\lambda)$ and $V(\tilde{b},\lambda)$. When $V(\tilde{a},\lambda)$ and $V(\tilde{b},\lambda)$ are equal, the ranking order of $\tilde{a}$ and $\tilde{b}$ depends on the relative position of $A(\tilde{a},\lambda)$ and $A(\tilde{b},\lambda)$. Thus, for the above example, according to Li and Nan's method, the ranking order of the three candidates is $A_3 \succ A_1 \succ A_2$ for any given $\lambda \in [0, 0.793]$.And if $\lambda \in (0.793, 1]$, $V(\tilde{S}_1,\lambda) > V(\tilde{S}_3,\lambda) > V(\tilde{S}_2,\lambda)$, hereby the ranking order of the three candidates is $A_1 \succ A_3 \succ A_2$. Furthermore, due to the fact that the value-index $V(\tilde{S}_1, 0.2) = 0.2298$ is smaller than $V(\tilde{S}_3, 0.2) = 0.2335$, according to Li and Nan's method, $A_3 \succ A_1$ for $\lambda = 0.2$ although the ambiguity-index $A(\tilde{S}_1, 0.2) = 0.0318$ is smaller than $A(\tilde{S}_3, 0.2) = 0.0374$. However, $\Delta(\tilde{S}_1, 0.2) = 0.198$ is bigger than $\Delta(\tilde{S}_3, 0.2) = 0.1961$. Hence, according to the proposed ranking method in this paper, we have $A_1 \succ A_3$. This analysis shows that the ambiguity-index plays an important role in the ranking order of TrIFNs.

(3) Comparison with Wang and Zhang' ranking method

There are some commonly-used methods which do not consider the maximum membership degrees and the minimum non-membership degrees, i.e., assume that $w_{\tilde{a}_{ij}} = 1$ and $u_{\tilde{a}_{ij}} = 0$. In this case, the TIFNs in Table 1 are reduced to the triangular fuzzy numbers. Thus, the above MADM problem with TIFNs is reduced to the MADM problem with triangular fuzzy numbers. Hereby, the weighted comprehensive values are obtained as $\hat{S}_1 = (0.592, 0.774, 0.910)$, $\hat{S}_2 = (0.769, 0.903, 1)$ and $\hat{S}_3 = (0.653, 0.849, 0.956)$. Using the existing ranking methods of fuzzy numbers, obviously, their ranking order is always $\hat{S}_2 > \hat{S}_3 > \hat{S}_1$, i.e., $A_2 \succ A_3 \succ A_1$, which is different from the results obtained by the proposed method considering maximum membership degrees and the minimum non-membership degrees. It shows that the maximum membership degrees and the minimum non-membership degrees are also very important in the ranking order of TrIFNs. Intuitively, the decision maker with different preference attitudes may have different choices. On the other hand, the method proposed by Wang and Zhang [2] transformed the ranking of TIFNs into that of interval numbers, which is difficult to be solved.

6. CONCLUSION

We discuss the value and ambiguity of a TrIFN, which are used to define the value-index and ambiguity-index of the TrIFN. Hereby the difference-index based ranking method is developed to rank TrIFNs and applied to solve MADM problems with TrIFNs. The proposed ranking method is a kind of two-index approaches, which aggregates both the value-index and ambiguity-index, and it takes into consideration the subjective attitude of the decision maker. And the proposed ranking method can be extended to rank more general IFNs in a straightforward manner due to the fact that the difference-index of a TrIFNs is not dependent on the form/shape of its membership and non-membership functions. Especially, the proposed ranking method has a natural appealing interpretation and possesses some good properties such as the linearity, which can be easily applied to real decision and optimization problems.

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